New Formulas Math 9B

Integral Calculus

APEX CALCULUS

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Differentiation Rules (From Math 9A - Written here to compare with integration rules below)

1. $\frac{d}{dx}(cx) = c$
2. $\frac{d}{dx}(u \pm v) = u' \pm v'$
3. $\frac{d}{dx}(u \cdot v) = uv' + u'v$
4. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}(u(v)) = u'(v)v'$
6. $\frac{d}{dx}(c) = 0$
7. $\frac{d}{dx}(x) = 1$
8. $\frac{d}{dx}(x^n) = nx^{n-1}$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$
11. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$
13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
16. $\frac{d}{dx}(\sec x) = \sec x \tan x$
17. $\frac{d}{dx}(\tan x) = \sec^2 x$
18. $\frac{d}{dx}(\cot x) = -\csc^2 x$
19. $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$
22. $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$
23. $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$
24. $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$
25. $\frac{d}{dx}(\cosh x) = \sinh x$
26. $\frac{d}{dx}(\sinh x) = \cosh x$
27. $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$
28. $\frac{d}{dx}(\text{sech}\, x) = -\text{sech}\, x \tanh x$
29. $\frac{d}{dx}(\text{csch}\, x) = -\text{csch}\, x \coth x$
30. $\frac{d}{dx}(\coth x) = -\text{csch}^2 x$
31. $\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}}$
32. $\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}}$
33. $\frac{d}{dx}\left(\text{sech}^{-1} x\right) = \frac{-1}{x\sqrt{1-x^2}}$
34. $\frac{d}{dx}\left(\text{csch}^{-1} x\right) = \frac{-1}{|x|\sqrt{1+x^2}}$
35. $\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$
36. $\frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}$

Integration Rules (New to Math 9B)

1. $\int c \cdot f(x)\, dx = c \int f(x)\, dx$
2. $\int f(x) \pm g(x)\, dx =$
   $\int f(x)\, dx \pm \int g(x)\, dx$
3. $\int 0\, dx = C$
4. $\int 1\, dx = x + C$
5. $\int x^n\, dx = \frac{1}{n+1}x^{n+1} + C,\ n \neq -1$
   $n \neq -1$
6. $\int e^x\, dx = e^x + C$
7. $\int a^x\, dx = \frac{1}{\ln a} \cdot a^x + C$
8. $\int \frac{1}{x}\, dx = \ln|x| + C$
9. $\int \cos x\, dx = \sin x + C$
10. $\int \sin x\, dx = -\cos x + C$
11. $\int \tan x\, dx = -\ln|\cos x| + C$
12. $\int \sec x\, dx = \ln|\sec x + \tan x| + C$
13. $\int \csc x\, dx = -\ln|\csc x + \cot x| + C$
14. $\int \cot x\, dx = \ln|\sin x| + C$
15. $\int \sec^2 x\, dx = \tan x + C$
16. $\int \csc^2 x\, dx = -\cot x + C$
17. $\int \sec x \tan x\, dx = \sec x + C$
18. $\int \csc x \cot x\, dx = -\csc x + C$
19. $\int \cos^2 x\, dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$
20. $\int \sin^2 x\, dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
21. $\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
22. $\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
23. $\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right) + C$
24. $\int \cosh x\, dx = \sinh x + C$
25. $\int \sinh x\, dx = \cosh x + C$
26. $\int \tanh x\, dx = \ln(\cosh x) + C$
27. $\int \coth x\, dx = \ln|\sinh x| + C$
28. $\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln\left|x + \sqrt{x^2-a^2}\right| + C$
29. $\int \frac{1}{\sqrt{x^2+a^2}}\, dx = \ln\left|x + \sqrt{x^2+a^2}\right| + C$
30. $\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2}\ln\left|\frac{a+x}{a-x}\right| + C$
31. $\int \frac{1}{x\sqrt{a^2-x^2}}\, dx = \frac{1}{a}\ln\left(\frac{x}{a+\sqrt{a^2-x^2}}\right) + C$
32. $\int \frac{1}{x\sqrt{x^2+a^2}}\, dx = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$

Formulas New To Math 9B:

Summation Formulas:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Definite integral is Limit of Riemann Sums:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \left[f(c_1) + f(c_2) + f(c_3) + f(c_4) + \ldots + f(c_{n\text{-}1}) + f(c_n) \right]$$

Example: Left-hand Riemann Sum $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \left[f(x_1) + f(x_2) + f(x_3) + \ldots + f(x_{n\text{-}1}) + f(x_n) \right]$

Example: RIght-hand Riemann Sum $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} \Delta x \left[f(x_2) + f(x_3) + f(x_4) + \ldots + f(x_n) + f(x_{n+1}) \right]$

FUNDAMENTAL THEOREM OF CALCULUS: Part 1 and Part 2

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x) \qquad \int_a^b F'(x)\,dx = F(b) - F(a)$$

Area Between Curves:

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx$$

Volume By Cross-Sectional Area:

$$V = \int_a^b A(x)\,dx$$

Arc Length:

$$L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$

	Disk Method	**Washer Method**	**Shell Method**
Horizontal Axis	$\pi \int_a^b R(x)^2\,dx$	$\pi \int_a^b \left(R(x)^2 - r(x)^2\right)dx$	$2\pi \int_c^d r(y)h(y)\,dy$
Vertical Axis	$\pi \int_c^d R(y)^2\,dy$	$\pi \int_c^d \left(R(y)^2 - r(y)^2\right)dy$	$2\pi \int_a^b r(x)h(x)\,dx$

Surfaces of Revolution with Respect to *x*-axis and *y*-axis:

$$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\,dx$$

(where $f(x) \geq 0$)

$$S = 2\pi \int_a^b x\sqrt{1 + f'(x)^2}\,dx$$

(where $a, b \geq 0$)